Tournaplexes and their Applications to Neuroscience

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(joint work with Ran Levi and Dejan Govc)

University of Aberdeen

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frontiers in Computational Neuroscience

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ORIGINAL RESEARCH ARTICLE

Front. Comput. Neurosci., 12 June 2017 | https://doi.org/10.3389/fncom.2017.00048

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Cliques of Neurons Bound into Cavities Provide a Missing Link between Structure and Function

Michael W. Reimann[1†], **Max Nolte**[1†], **Martina Scolamiero**[2], **Katharine Turner**[2], **Rodrigo Perin**[3], **Giuseppe Chindemi**[1], **Paweł Dłotko**[4‡], **Ran Levi**[5‡], **Kathryn Hess**[2*‡] and **Henry Markram**[1,3*‡]

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The lack of a formal link between neural network structure and its emergent function has hampered our understanding of how the brain processes information. We have now come closer to describing such a link by taking the direction of synaptic transmission into account, constructing graphs of a network that reflect the direction of information flow, and analyzing these directed graphs using algebraic topology. Applying this approach to a local network of neurons in the neocortex revealed a remarkably intricate and previously unseen topology of synaptic connectivity. The synaptic network contains an abundance of cliques of neurons bound into cavities that guide the emergence of correlated activity. In response to stimuli, correlated activity


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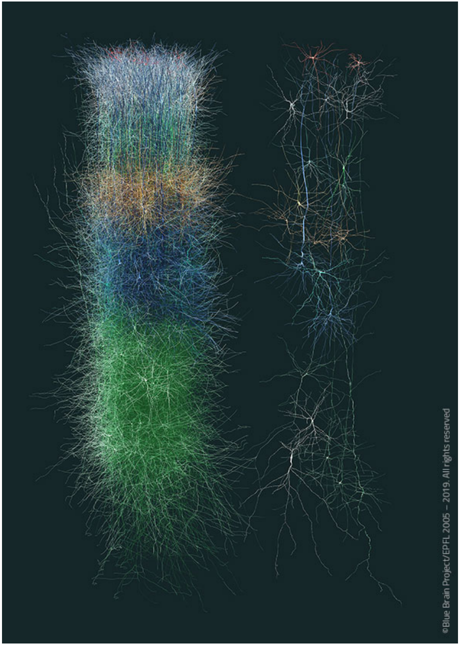
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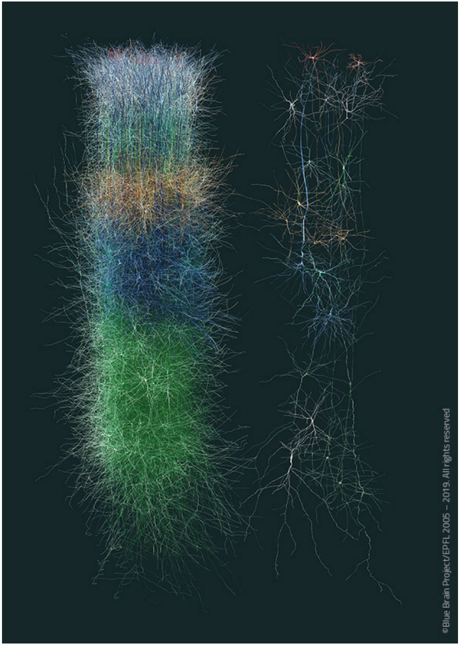
- $\sim$ 30 000 Neurons
- $\sim$ 8 000 000 Connections
- 6 Layers
- 54 Neuronal Types
- Functional Model



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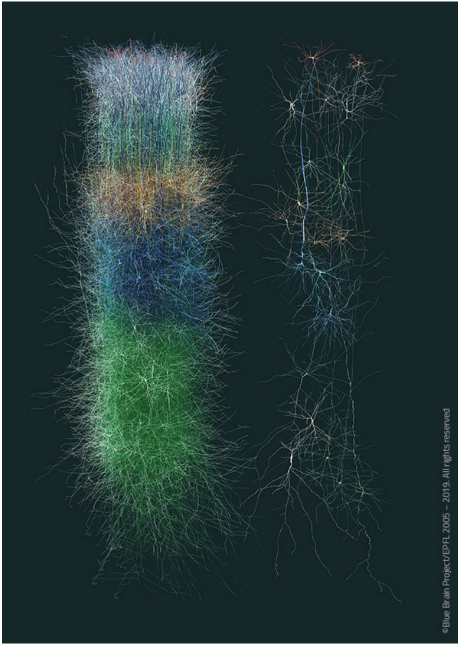
$$\downarrow$$

A Directed Graph



Goals

Study the structure of the brain graph

and

Stimulate the circuit and use the resulting activity to determine which stimulus was applied

Directed Flag Complex

Directed Graph $\longrightarrow$ Ordered Simplicial Complex

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Transitive Cliques $\longrightarrow$ Simplices

(i.e those containing no directed cycles)

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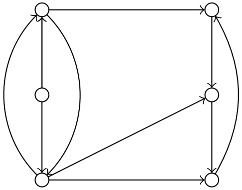

Directed Flag Complex

Directed Graph $\longrightarrow$ Ordered Simplicial Complex

Transitive Cliques $\longrightarrow$ Simplices

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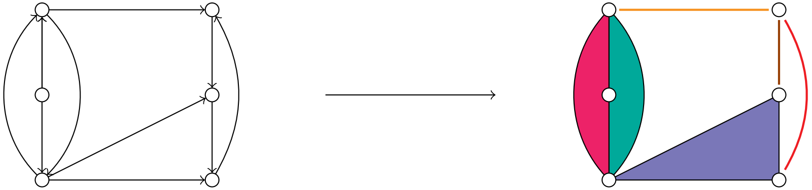
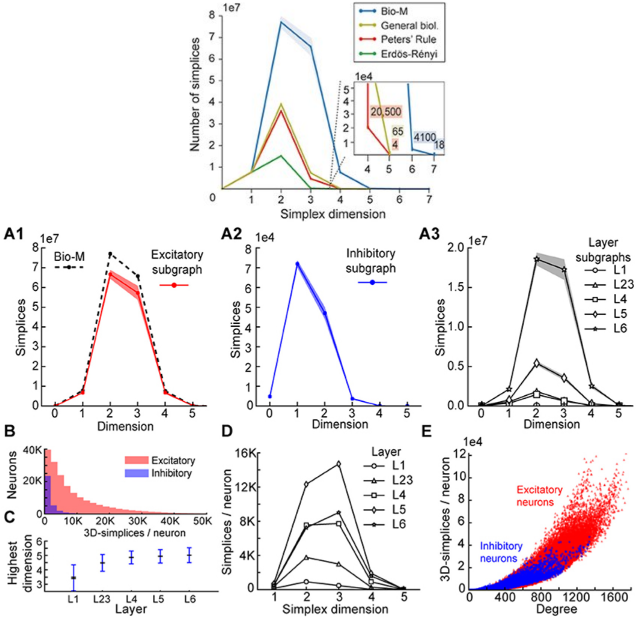
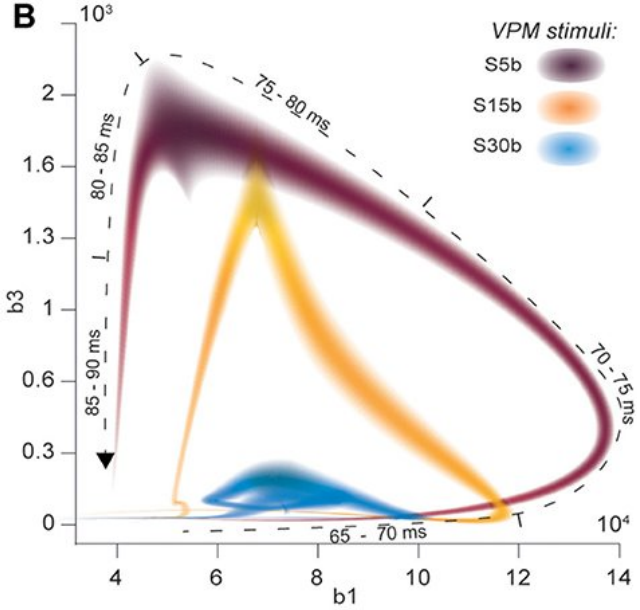


Structure



Number of simplices
Simplex dimension
Bio-M
General biol.
Peters' Rule
Erdös-Rényi
20,500
65
4
4100
18
A1
Simplices
Dimension
Excitatory subgraph
A2
Inhibitory subgraph
A3
Layer subgraphs
L1
L23
L4
L5
L6
B
Neurons
Excitatory
Inhibitory
3D-simplices / neuron
C
Highest dimension
Layer
D
Simplices / neuron
Simplex dimension
E
3D-simplices / neuron
Excitatory neurons
Inhibitory neurons
Degree

Function



B
10^3
VPM stimuli:
S5b
S15b
S30b
75 - 80 ms
80 - 85 ms
85 - 90 ms
70 - 75 ms
65 - 70 ms
2
1.6
1.3
1
0.6
0.3
0
b3
10^4
4
6
8
10
12
14
b1

Tournaplex

Directed Graph $\longrightarrow$ Geometric Realisation of Simplicial Set

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Cliques $\longrightarrow$ Simplices

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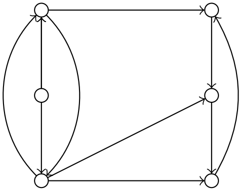

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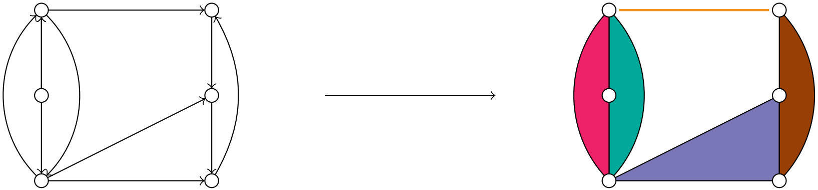

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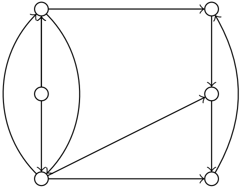
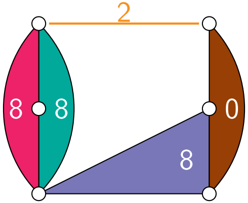
Directed Graph $\longrightarrow$ Geometric Realisation of Simplicial Set

Cliques $\longrightarrow$ Simplices



$\longrightarrow$



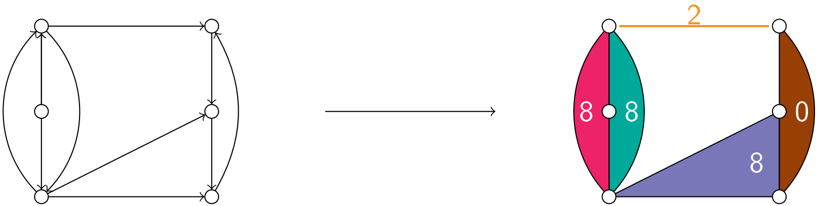
Directionality of clique σ:

$$dr_{local}(\sigma) = \sum_{v \in \sigma} (indegree_{\sigma}(v) - outdegree_{\sigma}(v))^2$$

Tournaplex

Directed Graph $\longrightarrow$ Geometric Realisation of Simplicial Set

Cliques $\longrightarrow$ Simplices

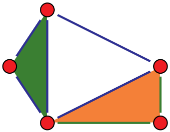


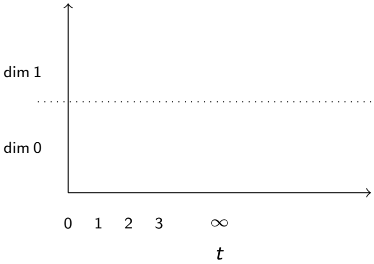
Directionality of clique σ:

$$dr_{local}(\sigma) = \sum_{v \in \sigma} (indegree_{\sigma}(v) - outdegree_{\sigma}(v))^2$$

$$dr_{global}(\sigma) = \sum_{v \in \sigma} (indegree_{G}(v) - outdegree_{G}(v))^2$$

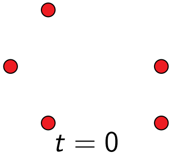
Persistent Homology

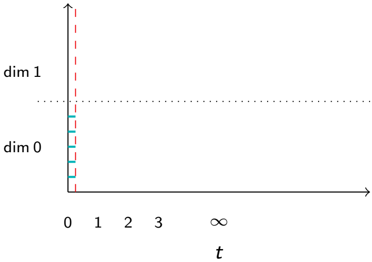




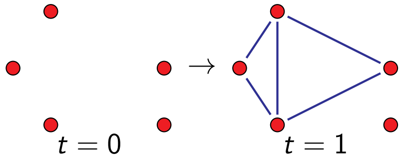
0
1
2
3

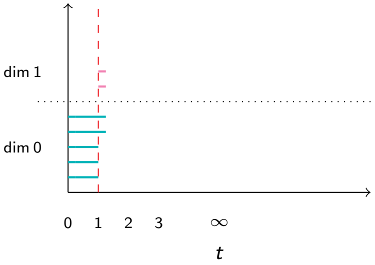
Persistent Homology



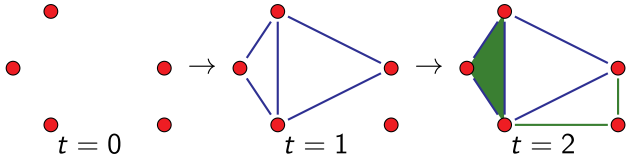


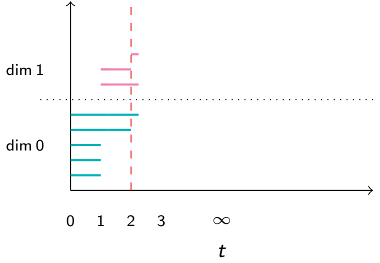
Persistent Homology





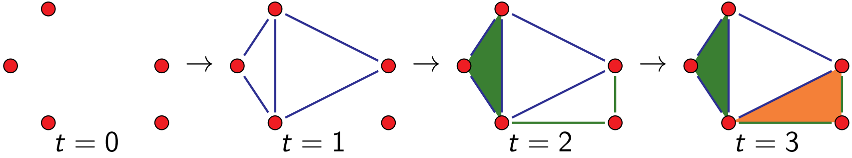
Persistent Homology

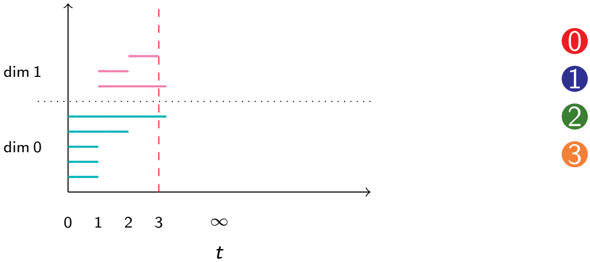




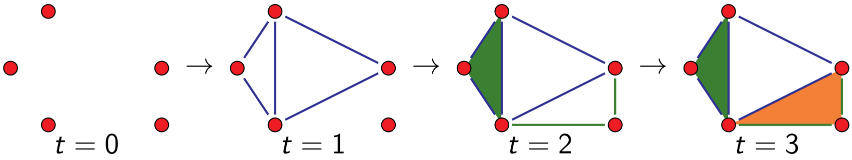
0
1
2
3

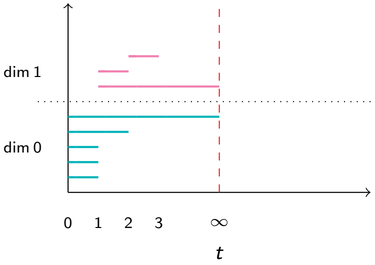
Persistent Homology





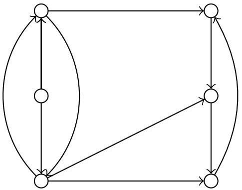
Persistent Homology

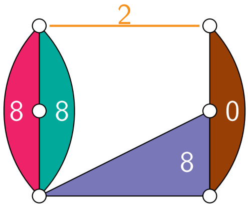




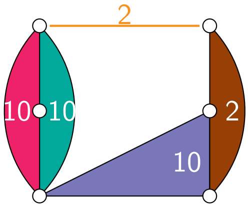
Directionality Filtrations

$$W(\sigma) = \sum_{v \in \sigma} (indegree_\sigma(v) - outdegree_\sigma(v))^2 + 2\binom{n}{3}$$





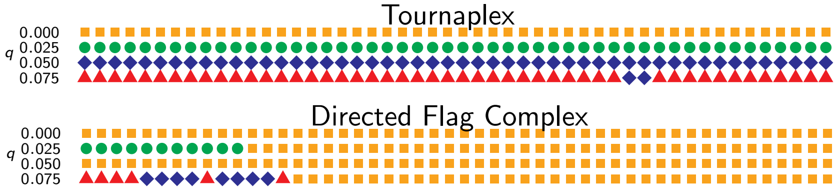
$dr_{local}(\sigma)$



$W(\sigma)$

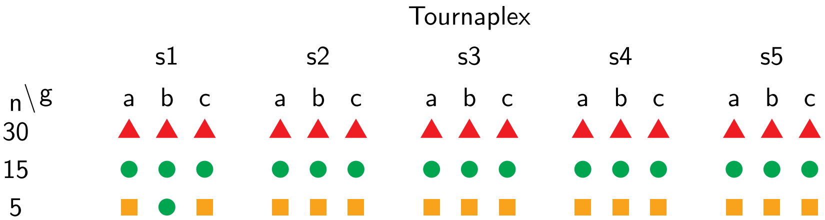
Distinguishing Graphs Using Tournaplex

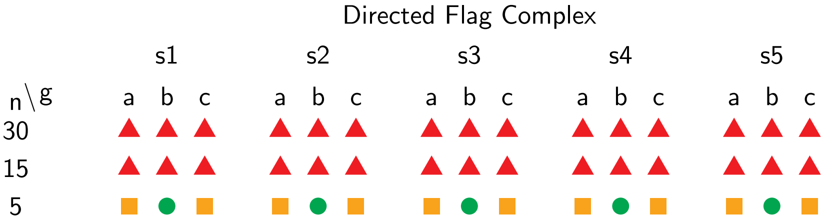
Data: 200 Erdős-Rényi graphs with edge $i \to j$ present with probability $\begin{cases} 0.25, & \text{if } i < j \\ q, & \text{if } i > j \end{cases}$



Distinguishing Stimuli

Data: 45 spike trains on Blue Brain model, 5 repetitions of 9 different stimuli





Thanks for Listening!

Many Thanks to the Blue Brain Project

Complexes of Tournaments, Directionality Filtrations and Persistent Homology
arXiv:2003.00324